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Pre- and Post-Selected Ensembles and Time-Symmetry in Quantum Mechanics

D.J. Miller

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Abstract It has recently been argued (Shimony, Erkenntnis 45:337, 1997) that time-symmetry does not hold for pre- and post-selected ensembles in quantum mechanics. That conclusion depends on what is meant by "time-symmetry" in relation to those types of ensembles. It is shown that on the conventional view of time-symmetry, pre- and post-selected ensembles are time-symmetric as was originally proposed.

Keywords Pre- and post-selected ensembles · Time-symmetry in quantum mechanics

1 Introduction

Quantum mechanics (QM) deals only with pre-selected ensembles, that is ensembles defined by a preparation outcome with a view to determining the probabilities of measurement outcomes in the future of the preparation event. Therefore QM is well-adapted to the intuitive concept of time, or more generally the A-theory of time, in which the past and the future have different status. The B-theory of time is an alternative, and counterintuitive, theory of time in which the past and the future have similar status. Modern physics is said by some (see [6] and [12] for discussions) to favour the "block universe view" which is a B-theory of time. If the B-theory or block universe view is correct, it seems more natural to consider ensembles which are pre-selected by the outcome of a preparation experiment and also post-selected by the outcome of a subsequent measurement with a view to determining the probabilities of the results of a measurement (or sequence of measurements) at intervening times.

Interest in pre- and post-selected ensembles (PPSEs) began with the work of Aharonov, Bergmann and Lebowitz (ABL) [4] and has been pursued mainly by Aharonov and co-workers [2, 3]. Sometimes QM for the PPSEs is referred to as the two-vector formalism [2]

D.J. Miller
School of Physics, University of New South Wales, Sydney, NSW 2052, Australia

D.J. Miller (✉)
Centre for Time, University of Sydney, Sydney, NSW 2006, Australia
e-mail: davmille@arts.usyd.edu.au

or time-symmetric quantum mechanics [3]. It is possible that ideas prompted by PPSEs have wider ramifications for QM [1, 9, 11].

Recently, Shimony has drawn two significant conclusions about PPSEs [14–16]. The first is that the "generalised state" [2] said to be determined by the "two vectors" specifying the initial and final conditions in a PPSE cannot be a regarded in the same way as the "state" determined by the initial conditions for the pre-selected (only) ensemble in QM. The main reason [14–16] is that the probabilities for the outcomes of the intermediate measurements of observables in a PPSE can depend on the method of their measurement, whereas the "state" in QM determines the probabilities of future measurement outcomes for any observable no matter how it is (faithfully) measured.

The second conclusion [14–16] concerns time-symmetry in relation to PPSEs. From the beginning, it had been thought [4] that, although QM has a built-in time asymmetry because it relies on initial conditions for subsequent properties, a PPSE could be regarded as time-symmetric for intermediate properties because it was selected by initial and final conditions. The two relevant conclusions of Aharonov et al. [4] are that (i) the laws of probability for PPSE's are time-symmetric and (ii) the laws for retrodiction are the same as the laws for prediction for PPSE's ([4], final paragraph, p. B1416). The latter conclusion is particularly significant because normally no probabilistic theory can have both predictive and retrodictive laws [7, 17, 18].

The formalism of Aharonov et al. [4] deals with measurements involving non-degenerate eigenvalues, although the extension to the degenerate case is obvious as pointed out in Aharonov et al. [4]. In Shimony [14, 15] and [16], the case of a degenerate eigenvalue is considered expressly and, furthermore, a case where the measurement apparatus may record a preferred basis for the degenerate eigenspace. This more general form of measurement still satisfies the condition that a repeated measurement yields the same eigenvalue [14]. Two specific examples are given in Shimony [14] which appear to show that the respective PPSE is not time-symmetric on either of two meanings of that term. Given that QM exhibits orthodox time-reversal symmetry [13, 19] and that a PPSE is defined in an apparently time-symmetric way, one would expect that a PPSE would preserve orthodox time-reversal symmetry. In the following, it is argued that a PPSE does satisfy orthodox time-reversal symmetry. This is illustrated with reference to the two examples in Shimony [14] which involve degenerate intermediate states. Cocke [5] has dealt with time-symmetry in relation to the ABL formalism for the non-degenerate case in terms of orthodox time-reversal symmetry and the following agrees with his results for that case.

2 Formalism for Pre- and Post-Selected Ensembles

Traditionally [2–4], a PPSE has been defined only in terms of states in the Hilbert space of the quantum system that is being measured. Nevertheless the Hilbert spaces of at least three pieces of measurement apparatus are also involved because a measurement needs to be performed in each of the pre-selection and post-selection steps to define the PPSE plus the measurement(s) involved at the intermediate time(s) to determine the properties that are calculated for the PPSE.

In many cases, expressly including the quantum systems involved in the various pieces of measurement apparatus would not change the final expressions for the probabilities of the intermediate measurement outcomes. In the present case, including the state of the intermediate measurement apparatus (IMA) is vital because Shimony deals with a degenerate eigenvalue for which, furthermore, the IMA registers two preferred bases. The state of the

IMA is included in the treatments in Shimony [14, 15] where, significantly for what follows, it is assumed that the state of the IMA should be reset when considering the reverse time direction. An issue in the following will be whether that is correct. Since re-setting measurement apparatus is an issue, it seems advisable to include the states of the sets of apparatus involved in the pre-selection and post-selection, as well as the IMA, in order to investigate what effect resetting or not resetting those states has on the final result. It turns out that the inclusion of the pre-selection and post-selection apparatus makes no material difference in the present instance because the pre- and post-selection involve non-degenerate states.

We will require the states of the quantum systems at the times of pre-selection t_a, intermediate measurement t_c and post-selection t_b. Since we will need to consider time evolution first in the forward direction and then in the backward direction, it will be necessary in the following to consider the various steps in the process more explicitly than is usually necessary. In the simplest possible case, the initial state of the system as a whole prior to the pre-selection can be specified by the state

$$|\Psi\rangle = |\psi\rangle \otimes |\gamma\rangle \otimes |\alpha\rangle \otimes |\beta\rangle \tag{1}$$

where $|\psi\rangle$, $|\gamma\rangle$, $|\alpha\rangle$ and $|\beta\rangle$ are the (assumed non-degenerate) states of the quantum system, the IMA, the pre-selection apparatus and the post-selection apparatus, respectively. In principle [16] it is necessary to deal with degeneracies and mixtures in relation to the initial state but the resulting expressions are not simple [4] and do not add anything new of a conceptual nature. The simple case of a non-degenerate initial state assumed here, along with similar assumptions at subsequent stages, are sufficient to deal with all the cases in the relevant literature.

Beginning with the above state, a practical means of carrying out the pre-selection at time t_a could involve a measurement for observable A of the quantum system with eigenvalues $|a_j\rangle$ in which it was possible to filter out all but one of the possible measurement outcomes. The latter step will be represented formally by the operator $\hat{R}(\alpha_i; t_a)$, or $\hat{R}(A; t_a)$ in the next section because the selection has to include $|\gamma\rangle$ to restore the original state in (2) since evolution in the backward time direction includes other states of the IMA. The pre-selected state at time t_a is

$$|A; t_a\rangle = |a_i\rangle \otimes |\gamma\rangle \otimes |\alpha_i\rangle \otimes |\beta\rangle. \tag{2}$$

Since the states at later times are mixtures, states will need to be expressed by density operators and in that form the state at t_a is

$$\hat{\rho}(t_a) = |A; t_a\rangle\langle A; t_a|. \tag{3}$$

The intermediate measurement is of observable C with possibly degenerate eigenvalues c^k and corresponding eigenstates $|c_l^k\rangle$ of the quantum system, where the $|c_l^k\rangle$ for $l \in \{\ldots\}_k$ span the subspace with eigenvalue c^k. In order to deal with all the cases of Shimony [14], it is necessary to consider a rather general form of the intermediate measurement. The system is prepared for the intermediate measurement during the time $t_c - \epsilon$ and t_c via a unitary interaction between the quantum system and the IMA and the measurement is completed at t_{c_+} by a non-unitary projection onto the possible outcome states according to the projection postulate. The projection can be taken to be instantaneous so t_{c_+} need not be distinguished from t_c except where necessary. We can assume no change occurs during the period t_a to $t_c - \epsilon$ due to the unitary time evolution $\hat{U}(t_c - \epsilon, t_a)$ which will therefore be written

$\hat{I}(t_c-\epsilon,t_a)$. The unitary interaction with the IMA in preparation for the measurement of C leads to the following state at time t_c

$$|A;t_c\rangle = \hat{U}(t_c,t_c-\epsilon)\hat{I}(t_c-\epsilon,t_a)|a_i\rangle \otimes |\gamma\rangle \otimes |\alpha_i\rangle \otimes |\beta\rangle \tag{4a}$$

$$= \sum_{k,l}\langle c_l^k|\hat{U}(t_c,t_a)|a_i\rangle \sum_m d_{lm}^k |c_m^k\rangle \otimes |\gamma_{lm}^k\rangle \otimes |\alpha_i\rangle \otimes |\beta\rangle \tag{4b}$$

where $\sum_m |d_{lm}^k|^2 = 1$ and the $|\gamma_{lm}^k\rangle$ for $l,m\in\{\ldots\}_k$ are all states of the measuring apparatus registering the experimental outcome with eigenvalue c^k. The evolution from t_a to t_c has no effect on (i.e. it is the identity operator in the Hilbert spaces of) the states of the pre- and post-selection apparatus.

The output states of the IMA $|\gamma_{lm}^k\rangle$ record both the projection of the quantum system onto one of a preferred set of basis states (index l above) and the subsequent projection of that state onto one of a second preferred set of basis states (index m above). For an eigenspace which is doubly-degenerate in spin, for example, one could imagine first passing the quantum system in state $|a_i\rangle$ through a Stern-Gerlach apparatus (SGA) and then passing each resulting beam through another SGA that is oriented independently. Each of the exit paths from the second SGA then correspond one of the $|\gamma_{lm}^k\rangle$ and the orientations of the first SGA and each of the second SGA's determine the probabilities $|d_{lm}^k|^2$ for each of the paths.

The measurement is completed at t_{c_+} when the states of the IMA are recorded. That process, which involves the projection postulate, will be represented formally by the operator $\hat{R}(\{\gamma_{lm}^k\};t_c)$ in the next subsection and the resulting state of the ensemble as a whole is the mixture

$$\hat{\rho}(t_c) = \sum_{k,l}|\langle c_l^k|\hat{U}(t_c,t_a)|a_i\rangle|^2 \sum_m |d_{lm}^k|^2 |c_m^k\rangle\langle c_m^k| \otimes |\gamma_{lm}^k\rangle\langle\gamma_{lm}^k| \otimes |\alpha_i\rangle\langle\alpha_i| \otimes |\beta\rangle\langle\beta|. \tag{5}$$

The operation of the projection postulate on the IMA states has no direct effect on (i.e. it is the identity operator in the Hilbert spaces of) the states of the quantum system and the pre- and post-selection apparatus.

Finally, the post-selection at time $t_b > t_c$ involves the measurement of observable B for the quantum system with (non-degenerate) eigenvalues $|b_i\rangle$ and the filtering out of all but one of the possible measurement outcomes. Thus the post-selection can be characterised by the evolution of each of the states $|c_m^k\rangle$ in the summation in (5) with the state $|\beta\rangle$ of the final measuring apparatus to form an entangled state $\hat{U}(t_b,t_c)|c_m^k\rangle \otimes |\beta\rangle \to \sum_i \langle b_i|\hat{U}(t_b,t_c)|c_m^k\rangle|b_i\rangle \otimes |\beta_i\rangle$. This time evolution has no effect on (i.e. it is the identity operator in the Hilbert spaces of) the states of the IMA and the pre-selection apparatus.

The final state of the PPSE is determined by selection on the measurement outcome $|\beta_j\rangle$ as a consequence of the projection postulate (this process will be represented formally by $\hat{R}(\beta_j;t_b)$ in the next section). The operation of the projection postulate on the states of the post-selection apparatus has no direct effect on (i.e. it is the identity operator in the Hilbert spaces of) the states of the quantum system, the IMA and the pre-selection apparatus. The final state of the PPSE as a whole is

$$\hat{\rho}(t_b) = \frac{1}{G}\sum_{k,l}\sum_m g_{lm}^k |b_j\rangle\langle b_j| \otimes |\gamma_{lm}^k\rangle\langle\gamma_{lm}^k| \otimes |\alpha_i\rangle\langle\alpha_i| \otimes |\beta_j\rangle\langle\beta_j| \tag{6}$$

where $g_{lm}^k = |\langle c_l^k|\hat{U}(t_c,t_a)|a_i\rangle|^2 |d_{lm}^k|^2 |\langle b_j|\hat{U}(t_b,t_c)|c_m^k\rangle|^2$ and $G=\sum_{k,l}\sum_m g_{lm}^k$. We will also require the state $\hat{\rho}(t_b|c_l^k\rangle)$ of the complete system at time t_b given the quantum system was recorded to be in the intermediate state $|c_l^k\rangle$ at t_c which is

$$\hat{\rho}(t_b|c_l^k\rangle) = \frac{1}{H}\sum_m h_{lm}^k |b_j\rangle\langle b_j| \otimes |\gamma_{lm}^k\rangle\langle\gamma_{lm}^k| \otimes |\alpha_i\rangle\langle\alpha_i| \otimes |\beta_j\rangle\langle\beta_j| \tag{7}$$

where $h_{lm}^k = |d_{lm}^k|^2 |\langle b_j|\hat{U}(t_b, t_c)|c_m^k\rangle|^2$ and $H = \sum_m h_{lm}^k$.

The set $\Gamma = \{\gamma_{lm}^k\}$ are distinct experimental outcomes determining the elementary events in the probability space. From Bayes' theorem (see [16] for a relevant discussion), the probability that the quantum system possessed the eigenvalue c^k at t_c, as measured by the set of IMA outcomes Γ, for the PPSE selected on the states $\hat{\rho}(t_a)$ and $\hat{\rho}(t_b)$ with time evolution in the forward direction is

$$\begin{aligned} &\mathrm{Prob}_f[c^k|\hat{\rho}(t_a), \Gamma, \hat{\rho}(t_b)] \\ &\quad = \frac{\mathrm{Prob}_f[b_j|\hat{\rho}(t_a), \Gamma, c^k]\mathrm{Prob}_f[c^k|\hat{\rho}(t_a), \Gamma]}{\mathrm{Prob}_f[b_j|\hat{\rho}(t_a), \Gamma]} &\text{(8a)} \\ &\quad = \frac{\sum_l \mathrm{Prob}_f[b_j|\hat{\rho}(t_a), \Gamma, c_l^k]\mathrm{Prob}_f[c_l^k|\hat{\rho}(t_a), \Gamma]}{\sum_k\sum_l \mathrm{Prob}_f[b_j|\hat{\rho}(t_a), \Gamma, c_l^k]\mathrm{Prob}_f[c_l^k|\hat{\rho}(t_a), \Gamma]}. &\text{(8b)} \end{aligned}$$

Each of the IMA measurement outcome states $\gamma_{l1}^k, \gamma_{l2}^k, \ldots, \gamma_{lm}^k$ constitute a measurement of the quantum system in the state c_l^k at t_c. Therefore, using (5),

$$\begin{aligned} \mathrm{Prob}[c_l^k|\hat{\rho}(t_a), \Gamma)] &= \sum_m \mathrm{Tr}(\hat{\rho}(t_c)|\gamma_{lm}^k\rangle\langle\gamma_{lm}^k|) &\text{(9a)} \\ &= |\langle c_l^k|\hat{U}(t_c, t_a)|a_i\rangle|^2. &\text{(9b)} \end{aligned}$$

The probability of the final state given the intermediate state c_l^k of the quantum system is, using the state in (6),

$$\begin{aligned} \mathrm{Prob}_f[b_j|\hat{\rho}(t_a), \Gamma, c_l^k] &= \mathrm{Tr}(\hat{\rho}(t_b|c_l^k\rangle)|b_j\rangle\langle b_j|) &\text{(10a)} \\ &= \frac{1}{H}\sum_m |d_{lm}^k|^2 |\langle b_j|\hat{U}(t_b, t_c)|c_m^k\rangle|^2. &\text{(10b)} \end{aligned}$$

Finally, from (8b),

$$\begin{aligned} &\mathrm{Prob}_f[c^k|\hat{\rho}(t_a), \Gamma, \hat{\rho}(t_b)] \\ &\quad = \frac{\sum_l |\langle c_l^k|\hat{U}(t_c, t_a)|a_i\rangle|^2 \sum_m |d_{lm}^k|^2 |\langle b_j|\hat{U}(t_b, t_c)|c_m^k\rangle|^2}{\sum_k\sum_l |\langle c_l^k|\hat{U}(t_c, t_a)|a_i\rangle|^2 \sum_m |d_{lm}^k|^2 |\langle b_j|\hat{U}(t_b, t_c)|c_m^k\rangle|^2}. \qquad \text{(11)} \end{aligned}$$

This is the generalisation of the usual ABL rule [2–4, 16] for the more complicated experimental situation considered in Shimony [14] which is being dealt with here.

3 Time-Symmetry and PPSEs

Firstly, it is necessary to be clear about the meaning of time-symmetry that is being used. There is not universal consensus on the meaning of time-symmetry either in classical physics or in quantum physics (for recent discussions and references to the literature, see [10] and [8] respectively). The orthodox position is that a system is time-symmetric if it satisfies motion reversal invariance [13, 19]. Motion reversal invariance is satisfied if a state is left unchanged by propagation forwards in time, followed by time reversal of the resulting state, propagation forwards in time again and a restoration of the original time sense [19]. That is,

$$\hat{\Theta}^{-1}\hat{U}(\Delta t)\hat{\Theta}\hat{U}(\Delta t) = \hat{I} \tag{12}$$

where $\hat{\Theta}$ is the time-reversal operator which produces the time-reversed state $|\tilde{X}\rangle$ of any state $|X\rangle$: $|\tilde{X}\rangle = \hat{\Theta}|X\rangle$ and $\hat{I}$ is the identity operator in the relevant Hilbert space. This requires that

$$\hat{\Theta}^{-1}\hat{U}(\Delta t)\hat{\Theta} = \hat{U}^{\dagger}(\Delta t) \tag{13}$$

which, from the form of the time-evolution operator $\hat{U}(\Delta t) = \exp(-i\hat{H}\Delta t)$ for $\hat{H}$ constant in time and $\hat{H} = \hat{H}^{\dagger}$, is true if the Hamiltonian satisfies

$$\hat{H} = \hat{\Theta}^{-1}\hat{H}\hat{\Theta}. \tag{14}$$

For the case when the Hamiltonian is not constant in time, for example the present case when a sequence of different Hamiltonians each constant in time are applied to the system, one can break up the time evolution into intervals Δt_n over each of which the applicable Hamiltonians H_n is constant in time so that

$$\hat{U}(\Delta t) = \hat{U}_1(\Delta t_1)\hat{U}_2(\Delta t_2)\dots\hat{U}_n(\Delta t_n) \tag{15}$$

where $\sum_n \Delta t_n = \Delta t$. Orthodox time reversal invariance, i.e. motion reversal invariance, in this case requires that the same state is obtained after propagation forward in time by the sequence of Hamiltonians, time-reversal of the resulting state, propagation forward in time by the sequence of Hamiltonians in reverse order followed by time-reversal of the resulting state. That is,

$$\hat{\Theta}^{-1}\hat{U}_n(\Delta t_n)\dots\hat{U}_2(\Delta t_2)\hat{U}_1(\Delta t_1)\hat{\Theta}\hat{U}_1(\Delta t_1)\hat{U}_2(\Delta t_2)\dots\hat{U}_n(\Delta t_n) = \hat{I} \tag{16}$$

and this follows from the form of the time-evolution operator if (14) is satisfied because

$$\begin{aligned}&\hat{\Theta}^{-1}\hat{U}_n(\Delta t_n)\dots\hat{U}_2(\Delta t_2)\hat{U}_1(\Delta t_1)\hat{\Theta}\\ &\quad= \hat{U}_n^{\dagger}(\Delta t_n)\dots\hat{U}_2^{\dagger}(\Delta t_2)\hat{U}_1^{\dagger}(\Delta t_1) = \hat{U}^{\dagger}(\Delta t).\end{aligned} \tag{17}$$

Another, equivalent, approach to time symmetry in the orthodox sense is to note that $\hat{U}^{\dagger}(\Delta t) = \hat{U}(-\Delta t)$, which again follows from the form of the time evolution operator and $\hat{H}^{\dagger} = \hat{H}$. Therefore (using the second equality in (17)), the original state is always regained by applying a series of time independent Hamiltonians in the forward time direction, and then applying to the resulting, final state, the reverse sequence of Hamiltonians in the reverse time direction. Orthodox time reversal symmetry requires that the original state is obtained by applying a series of time independent Hamiltonians in the forward time direction, and then applying to the resulting, final state, the reverse sequence of *time-reversed* Hamiltonians in the reverse time direction. Therefore, once again, orthodox time reversal symmetry follows if (14) is satisfied.

The result that time symmetry is ensured by (14) has assumed that the time evolutions are unitary. In the case of PPSE's, measurements are required to define the PPSE and these involve non-unitary projections which also make the theory probabilistic rather than deterministic. Time-reversal invariance for a probabilistic theory, for example quantum mechanics including measurements, requires due consideration [7]. Furthermore the above criteria which involve re-gaining an initial state after forward and backward time evolutions to and from a final state is not suitable for PPSE's because the PPSE is *selected* on the initial and final states. The criterion for time symmetry of PPSE's must involve their main feature, namely the probabilities of the intermediate properties of the quantum system or, equivalently, the probabilities of the intermediate measurement outcomes. The criterion used previously [3–5, 14, 15] is that the probabilities of the intermediate properties be the same when they are calculated in the forward and the backward time directions. The question then becomes how the probabilities are to be calculated in the backward time direction.

In the present case, for evolution in the forward direction, the final state $\hat{\rho}(t_b)$ in (7) is obtained from $\hat{\rho}(t_b) = \hat{S}(t_b, t_a)\hat{\rho}(t_a)\hat{S}^\dagger(t_b, t_a)$, where the operator sequence (read from right to left)

$$\hat{S}(t_b, t_a) = \hat{R}(\beta_j; t_b)\hat{U}(t_b, t_c)\hat{R}(\{\gamma^k_{lm}\}; t_{c+})\hat{U}(t_c, t_c - \epsilon)\hat{I}(t_c - \epsilon, t_a)\hat{R}(A; t_a). \tag{18}$$

From the earlier consideration in this subsection, it would seem that the test of whether the PPSE satisfies orthodox time-reversal invariance is whether the same probability of obtaining the eigenvalue c^k of $\hat{C}$ for the intermediate measurement results from (i) the application to $\hat{\rho}(t_a)$ of the sequence in (18) and from (ii) the application to $\hat{\rho}(t_b)$ of the sequence

$$\begin{aligned}\hat{S}(t_a, t_b) &= \hat{S}^\dagger(t_b, t_a)\\ &= \hat{R}(A; t_a)\hat{I}(t_a, t_c - \epsilon)\hat{U}(t_c - \epsilon, t_c)\hat{R}(\{\gamma^k_{lm}\}; t_{c+})\hat{U}(t_c, t_b)\hat{R}(\beta_j; t_b)\end{aligned} \tag{19}$$

i.e. the same sequence of interactions applied in reverse order and in the reverse time direction. In obtaining (19), we have used $\hat{U}^\dagger(t_1, t_2) = \hat{U}(t_2, t_1)$ and $\hat{R}^\dagger(x; t_x) = \hat{R}(x; t_x)$.

An alternative approach to time symmetry for a PPSE is adopted in Shimony [14–16]. To distinguish it from orthodox time symmetry, the criterion could be termed *time symmetry for pre-and post-selection (PPS)* and the PPSE would be said to time symmetric for PPS if the same probability for the intermediate eigenvalue c^k of $\hat{C}$ is obtained from (i) pre-selecting the quantum system in a state $|a_i\rangle$ with the IMA in the measurement-ready state $|\gamma\rangle$, measuring $\hat{C}$ in the forward time direction and post-selecting on a state $|b_j\rangle$ and from (ii) pre-selecting the quantum system in a state $|b_j\rangle$ with the IMA in the measurement-ready state $|\gamma\rangle$, measuring $\hat{C}$ in the reverse time direction and post-selecting on a state $|a_i\rangle$. If a PPSE could be always properly specified solely in terms of the quantum system that is being measured, then re-setting or not re-setting the IMA should lead to the same PPSE and hence the same probabilities. However it is successfully argued in Shimony [14] that the specification of the PPSE involves the intermediate measurement. In that case re-setting the IMA becomes a crucial point. Time symmetry for PPS is obviously a significant property of PPSE's and Shimony [14, 15] has shown that although it applies to PPSE's in cases considered previously it does not apply to PPSE's in a more general case. Nevertheless, the property here called time symmetry for PPS does not correspond to orthodox time symmetry and it does not follow from a demonstration that PPSE's are not time symmetrical for PPS that PPSE's do not satisfy orthodox time reversal symmetry. This is because there are two significant differences between the two concepts.

The first is that, for time symmetry for PPS, the final state of the IMA is re-set to its measurement ready condition, that is the final state is changed to (in the notation of the previous subsection)

$$\hat{\rho}'(t_b) = |b_j\rangle \otimes |\gamma\rangle \tag{20}$$

rather than (7) for the purposes of consideration in the reverse time direction. The omission from explicit consideration of the last two terms of (7), $|\alpha_i\rangle \otimes |\beta_j\rangle$, in (20) makes no material difference but the re-setting of the IMA to the original state $|\gamma\rangle$ is significant. The second difference is that the sequence applied to $\hat{\rho}'(t_b)$ for the reverse time direction calculation is taken to be

$$\hat{S}'(t_a, t_b) = \hat{R}(A, t_a)\hat{I}(t_a, t_c - \epsilon)\hat{R}(\{\gamma^k_{lm}\}, t_c - \epsilon)\hat{U}(t_c - \epsilon, t_c) \tag{21}$$

instead of (19) above. The omission from (21) of $\hat{U}(t_c, t_b)\hat{R}(\beta_j, t_b)$ in (19) makes no material difference because the former evolution is only necessary if the final measurement apparatus is expressly included. The significant point is the replacement of $\hat{U}(t_c, t_c - \epsilon)\hat{R}(\{\gamma^k_{lm}\}, t_{c_+})$ in (19) by $\hat{R}(\{\gamma^k_{lm}\}, t_c - \epsilon)\hat{U}(t_c - \epsilon, t_c)$. This change in the time of measurement from t_c in the forward direction to $t_c - \epsilon$ in the reverse time direction is consistent with, and required by, re-setting the state of the IMA to $|\gamma\rangle$ at t_b but it is a significant physical alteration of the sequence of events in the reverse time direction which results in a departure from orthodox time reversal invariance.

It has been concluded in the past [4, 5] that if the orthodox approach to time-reversal is adopted, the calculation of the intermediate probabilities for a PPSE is time symmetric. Nevertheless, those demonstrations did not expressly consider the type of intermediate measurement considered in Shimony [14, 15] and the intermediate measurement and the pre- and post-selections do involve non-unitary projections of states, so it seems worthwhile to show expressly that both of the counterexamples from Shimony [14] are indeed time-symmetric on the orthodox view.

It is worth noting that it is said [16, p. 230] that the propagation backward through the interval of the intermediate measurement (as defined here) t_c to $t_c - \epsilon$ "is illegitimate from the standpoint of standard quantum mechanics, since the system interacts with the measuring apparatus" during this interval. The same comment could be made for propagation backward through the time interval of the final measurement t_b to t_c. However the interaction between the measuring apparatus and the quantum system is unitary; it is the second step of the measurement, which projects the quantum system plus apparatus onto the observed state which is non-unitary. Therefore, provided the Hamiltonian describing the interaction is time symmetric, i.e. $\hat{H} = \hat{\Theta}^{-1}\hat{H}\hat{\Theta}$, there is no reason why the interaction step with the IMA cannot be considered in the reverse direction of time. Of course, the original state (in the sense of forward-time propagation) will not be restored by the backward evolution of the projected state. Nevertheless, it is now shown that conventional time-symmetry ensures that the same PPSE and intermediate probabilities result from consideration in both time directions.

3.1 Consideration of the First Counter-Example of Shimony [14]

In our notation, the example involves the intermediate measurement of observable C which has a non-degenerate eigenvalue c^0 and corresponding eigenstate $|c^0_0\rangle$ and a doubly-degenerate eigenvalue c^1 with the corresponding eigenspace spanned by $|c^1_1\rangle$ and $|c^1_2\rangle$. The pre-selection is implemented by projection of the quantum system at time t_a leading to the following initial state for the quantum system and IMA is

$$|A; t_a\rangle = |a_i\rangle \otimes |\gamma\rangle \otimes |\alpha_i\rangle \otimes |\beta\rangle \tag{22a}$$

$$= \frac{1}{\sqrt{2}}(|c^0_0\rangle + |c^1_1\rangle) \otimes |\gamma\rangle \otimes |\alpha_i\rangle \otimes |\beta\rangle \tag{22b}$$

and

$$\hat{\rho}(t_a) = |A; t_a\rangle\langle A; t_a|.$$

In our notation, the Hamiltonian (h_{tot} in (17) of Shimony [14]) describing the interaction between the quantum system and the IMA

$$\hat{h}_{\text{tot}} = g \sum_k \sum_{l,m} (d^k_{lm} |c^k_m\rangle\langle c^k_l| \otimes |\gamma^k_{lm}\rangle\langle\gamma| + (d^k_{lm})^* |c^k_l\rangle\langle c^k_m| \otimes |\gamma\rangle\langle\gamma^k_{lm}|) + g' \hat{I}' \tag{23}$$

where g, g' are real and $\hat{I}'$ is the identity operator in the Hilbert space spanned by the states which are do not appear in the summation term. The eigenvalues of $\hat{h}_{\text{tot}}$ are $E = 0$ and $E = \pm g$ with corresponding energy eigenstates

$$|\tau^1_1\rangle = -(d^1_{12})^* |c^1_1\rangle \otimes |\gamma^1_{11}\rangle + (d^1_{11})^* |c^1_2\rangle \otimes |\gamma^1_{12}\rangle \tag{24a}$$

$$|\tau^1_2\rangle = (d^1_{22})^* |c^1_1\rangle \otimes |\gamma^1_{21}\rangle - (d^1_{21})^* |c^1_2\rangle \otimes |\gamma^1_{22}\rangle \tag{24b}$$

for $E = 0$ and

$$|\sigma^0_\pm\rangle = \frac{1}{\sqrt{2}}[|c^0_0\rangle \otimes |\gamma\rangle \pm |c^0_0\rangle \otimes |\gamma^0_{00}\rangle] \tag{25a}$$

$$|\sigma^1_{1,\pm}\rangle = \frac{1}{\sqrt{2}}[|c^1_1\rangle \otimes |\gamma\rangle \pm (d^1_{11}|c^1_1\rangle \otimes |\gamma^1_{11}\rangle + d^1_{12}|c^1_2\rangle \otimes |\gamma^1_{12}\rangle)] \tag{25b}$$

$$|\sigma^1_{2,\pm}\rangle = \frac{1}{\sqrt{2}}[|c^1_2\rangle \otimes |\gamma\rangle \pm (d^1_{21}|c^1_1\rangle \otimes |\gamma^1_{21}\rangle + d^1_{22}|c^1_2\rangle \otimes |\gamma^1_{22}\rangle)] \tag{25c}$$

for $E = \pm g$.

For the chosen period of interaction $\epsilon = \pi/2g$, the time evolution operator $\hat{U}(t_c, t_c - \epsilon) = e^{-i\hat{h}_{\text{tot}}\epsilon} = e^{-iE\epsilon}$ leaves the $E = 0$ states unchanged and causes the $E = \pm g$ states to be multiplied by $\mp i$ respectively. In terms of the energy eigenstates, the initial state (cf. (2)) is

$$|A; t_a\rangle = \frac{1}{2}(|\sigma^0_+\rangle + |\sigma^0_-\rangle + |\sigma^1_{1,+}\rangle + |\sigma^1_{1,-}\rangle) \otimes |\alpha_i\rangle \otimes |\beta\rangle \tag{26}$$

so the unitary interaction between the quantum system and the IMA between $t_c - \epsilon$ and t_c (with $\hat{U}(t_c - \epsilon, t_a) = \hat{I}(t_c - \epsilon, t_a)$) causes the evolution (in the forward-time direction)

$$\begin{aligned}\hat{U}(t_c, t_a)|A; t_a\rangle = |C; t_a\rangle &= -\frac{i}{2}(|\sigma^0_+\rangle - |\sigma^0_-\rangle + |\sigma^1_{1,+}\rangle - |\sigma^1_{1,-}\rangle) \otimes |\alpha_i\rangle \otimes |\beta\rangle \\ &= -\frac{i}{\sqrt{2}}[|c^0_0\rangle \otimes |\gamma^0_{00}\rangle - i(d^1_{11}|c^1_1\rangle \otimes |\gamma^1_{11}\rangle + d^1_{12}|c^1_2\rangle \otimes |\gamma^1_{12}\rangle) \otimes |\alpha_i\rangle \otimes |\beta\rangle].\end{aligned} \tag{27}$$

After the projection at t_{c+}, the state is (cf. (5))

$$\begin{aligned}\hat{\rho}(t_c) = \frac{1}{2}(&|c^0_0\rangle\langle c^0_0| \otimes |\gamma^0_{00}\rangle\langle\gamma^0_{00}| \\ &+ \sum_m |d^1_{1m}|^2 |c^1_m\rangle\langle c^1_m| \otimes |\gamma^1_{1m}\rangle\langle\gamma^1_{1m}| \otimes |\alpha_i\rangle\langle\alpha_i| \otimes |\beta\rangle\langle\beta|.\end{aligned} \tag{28}$$

The post-selection is implemented at time $t_b > t_c$ to obtain the state of the quantum system

$$|b_j\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle + |c_2^1\rangle). \tag{29}$$

This can be accomplished by subjecting the quantum system during the time interval t_c to t_b to an interaction which results in a suitable pre-measurement state. A specific form of the corresponding unitary time evolution operator $\hat{U}(t_b, t_c)$ is required so that its effect in the backward time direction can be calculated later. A suitable expression is

$$\begin{aligned}\hat{U}(t_b, t_c) = {} & |b_j\rangle\langle b_j| \otimes (|\beta_j\rangle\langle\beta| + |\beta\rangle\langle\beta_{j'}| + |\beta_{j'}\rangle\langle\beta_{j''}| + |\beta_{j''}\rangle\langle\beta_j|) \\ & + |b_j^\perp\rangle\langle b_j^\perp| \otimes (|\beta_{j'}\rangle\langle\beta| + |\beta\rangle\langle\beta_{j''}| + |\beta_{j''}\rangle\langle\beta_j| + |\beta_j\rangle\langle\beta_{j'}|) \\ & + |c_1^1\rangle\langle c_1^1| \otimes (|\beta_{j''}\rangle\langle\beta| + |\beta\rangle\langle\beta_j| + |\beta_j\rangle\langle\beta_{j'}| + |\beta_{j'}\rangle\langle\beta_{j''}|)\end{aligned} \tag{30}$$

where $|b_j^\perp\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle - |c_2^1\rangle)$ and $|\beta\rangle, |\beta_j\rangle, |\beta_{j'}\rangle$ and $|\beta_{j''}\rangle$ span the Hilbert space of the detector. The state in (29) is then selected by the detector registering the state $|\beta_j\rangle$ which is the effect of $\hat{R}(\beta_j; t_b)$ in (18). The final state of the PPSE is then

$$\hat{\rho}(t_b) = \frac{1}{N}|b_j\rangle\langle b_j| \otimes (|\gamma_{00}^0\rangle\langle\gamma_{00}^0| + |d_{12}^1|^2|\gamma_{12}^1\rangle\langle\gamma_{12}^1|) \otimes |\alpha_i\rangle\langle\alpha_i| \otimes |\beta_j\rangle\langle\beta_j| \tag{31}$$

where $N = 1 + |d_{12}^1|^2$.

Using the states given in (22), (28) and (31), the probability from (8b) that in the forward time direction that the intermediate measurement yielded the eigenstate c^1 is

$$\text{Prob}_f[c^1|\hat{\rho}(t_a), \Gamma, \hat{\rho}(t_b)] = \frac{|d_{12}^1|^2}{1 + |d_{12}^1|^2} \tag{32}$$

which, in our notation, is in agreement with (26a) of Shimony [14].

As discussed in Sect. 3, we have proposed that to confirm that $\text{Prob}[c^1|\hat{\rho}(t_a), \Gamma, \hat{\rho}(t_b)]$ is time-symmetric, one should take the final state of the quantum system plus apparatus ((31) in this case), evolve it backward in time using (19) and select on the original state in (22). By the reasoning that led to (8), in the reverse time direction the probability is

$$\text{Prob}_r[c^1|\hat{\rho}(t_a), \Gamma, \hat{\rho}(t_b)] = \frac{\sum_l \text{Prob}_r[a_i|c_l^1, \Gamma, \hat{\rho}(t_b)]\text{Prob}_r[c_l^1|\Gamma, \hat{\rho}(t_b)]}{\sum_k \sum_l \text{Prob}_r[a_i|c_l^k, \Gamma, \hat{\rho}(t_b)]\text{Prob}_r[c_l^k|\Gamma, \hat{\rho}(t_b)]}. \tag{33}$$

Thus we begin with the state $\hat{\rho}(t_b)$ in (31) at time t_b and evolve it back through the interval t_b to t_c by applying $\hat{U}(t_c, t_b) = \hat{U}^\dagger(t_b, t_c)$ with $\hat{U}(t_b, t_c)$ given in (30). That evolution results in the state

$$\hat{\rho}_r(t_c) = \frac{1}{N}|b_j\rangle\langle b_j| \otimes (|\gamma_{00}^0\rangle\langle\gamma_{00}^0| + |d_{12}^1|^2|\gamma_{12}^1\rangle\langle\gamma_{12}^1|) \otimes |\alpha_i\rangle\langle\alpha_i| \otimes |\beta\rangle\langle\beta|. \tag{34}$$

From (19), the next operation is projection $\hat{R}(\{\gamma_{lm}^k\}, t_{c+})$ which leaves (34) unchanged. Next, the interaction between the quantum system and the IMA given in (23), causes the $\pm g$ energy states to be multiplied by $\pm i$ for the evolution $\hat{U}(t_c - \epsilon, t_c)$ in the backward time direction. To calculate the effect of $\hat{U}(t_c - \epsilon, t_c)$, the following results are needed.

$$\hat{U}(t_c-\epsilon,t_c)|b_j\rangle\otimes|\gamma^0_{00}\rangle$$
$$=\hat{U}(t_c-\epsilon,t_c)\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|\sigma^0_+\rangle-|\sigma^0_-\rangle)+|c^1_2\rangle\otimes|\gamma^0_{00}\rangle\right]$$
$$=\frac{1}{\sqrt{2}}[i|c^0_0\rangle\otimes|\gamma\rangle+\exp(ig'\epsilon)|c^1_2\rangle\otimes|\gamma^0_{00}\rangle] \tag{35a}$$
$$\hat{U}(t_c-\epsilon,t_c)|b_j\rangle\otimes|\gamma^1_{12}\rangle$$
$$=\hat{U}(t_c-\epsilon,t_c)\frac{1}{\sqrt{2}}\left[|c^0_0\rangle\otimes|\gamma^1_{12}\rangle+\frac{i}{\sqrt{2}}(d^1_{12})^*(|\sigma^1_+\rangle-|\sigma^1_-\rangle)+d^1_{11}|\tau^1_1\rangle\right]$$
$$=\frac{1}{\sqrt{2}}[\exp(ig'\epsilon)|c^0_0\rangle\otimes|\gamma^1_{12}\rangle+i(d^1_{12})^*|c^1_1\rangle\otimes|\gamma\rangle+d^1_{11}|\tau^1_1\rangle]. \tag{35b}$$

The final step is the selection of the original initial state given in (22) and this results from the operation of the remaining terms in (19), $\hat{R}(A,t_a)\hat{I}(t_a,t_c-\epsilon)$, on $\hat{U}(t_c-\epsilon,t_c)\hat{\rho}_r(t_c)\hat{U}^\dagger(t_c-\epsilon,t_c)$ using (35). This leads to the final state in the reverse direction $\hat{\rho}_r(t_a)=|A\rangle\langle A|$. We also need the states of the complete system $\hat{\rho}_r(t_b|c^k_l\rangle)$ when the quantum system was recorded to be in the intermediate state $|c^k_l\rangle$ which can be found using (34) and (35). The terms in (33) can now be calculated:

$$\mathrm{Prob}_r[c^k_l|\Gamma,\hat{\rho}(t_b)]=\mathrm{Tr}(\hat{\rho}_r(t_c)|c^k_l\rangle\langle c^k_l|) \tag{36}$$

with $\hat{\rho}_r(t_c)$ given in (34) and

$$\mathrm{Prob}_r[a_i|c^k_l,\Gamma,\hat{\rho}(t_b)]=\mathrm{Tr}(\hat{\rho}_r(t_a)|A;t_a\rangle\langle A;t_a|) \tag{37}$$

where $\hat{\rho}_r(t_a)=\hat{I}(t_a,t_c-\epsilon)\hat{\rho}_r(t_c-\epsilon)=\hat{\rho}_r(t_c-\epsilon)$, $\hat{\rho}_r(t_c-\epsilon)$ is the state in (35) and $|A:t_a\rangle$ is the state in (22a). Therefore

$$\mathrm{Prob}_r[c^1|\hat{\rho}(t_a),\Gamma,\hat{\rho}(t_b)]=\frac{|d^1_{12}|^2}{1+|d^1_{12}|^2}. \tag{38}$$

Comparison with (32) shows that the same probability is obtained for the intermediate state from calculation in either the forward or backward time direction if orthodox time reversal symmetry is used.

3.2 Consideration of the Second Counter-Example of Shimony [14]

A second counter-example to time symmetry for a PPSE is given in Appendix A of Shimony [14]. It addresses the suggestion that time symmetry would mean that the probabilities for a PPSE defined by an evolution of an initial state at time t_a forward in time to a final state at t_b should be the same when the initial and final states are interchanged (retaining the same forward evolution from t_a to t_b). The counter-example shows that this criterion for time symmetry does not hold even for the simplest case of an intermediate measurement involving only non-degenerate eigenvalues. As discussed in Sect. 3, the criterion of orthodox time-reversal which is closest to the above criterion would require that the probabilities were the same when the initial and final states are interchanged *and* the initial and final states were time-reversed. The purpose here is to confirm that this last criterion, corresponding to orthodox time-reversal symmetry, is satisfied by the counter-example.

In the counter-example, the quantum system is in a four-dimensional Hilbert space with one non-degenerate eigenstate $|c_0^0\rangle$ with eigenvalue c^0 and triply degenerate eigenstates $|c_i^1\rangle$ $(i = 1, 2, 3)$ with eigenvalue c^1. As for the first example, the pre- and post-selected states of the quantum system are

$$|a_i\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle + |c_1^1\rangle) \tag{39}$$

and

$$|b_j\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle + |c_2^1\rangle). \tag{40}$$

It is assumed that the quantum system is subject to the same time-evolution operator $\hat{U}(t_c, t_a) = \hat{U}(t_b, t_c) = \hat{U}$ both between t_a and t_c and between t_c and t_b. In the $|c_0^0\rangle$, $|c_i^1\rangle$ basis, $\hat{U}$ is defined to be

$$\hat{U} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{41}$$

We first calculate the probabilities in the forward time direction. The intermediate measurement simply distinguishes between the eigenvalues c^0 and c^1 so the IMA states are simply $|\gamma^0\rangle$ and $|\gamma^1\rangle$. Thus the state of the quantum system and the IMA at time t_c due to the evolution between t_a and t_c, given by $\hat{U}$ of (44), followed by the interaction between the quantum system and the IMA is

$$|A, t_c\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle \otimes |\gamma^0\rangle + |c_2^1\rangle \otimes |\gamma^1\rangle) \otimes |\alpha_i\rangle \otimes |\beta\rangle. \tag{42}$$

After the evolution between t_c and t_b, again given by $\hat{U}$ of (44), the state becomes

$$|A; t_b\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle \otimes |\gamma^0\rangle + |c_3^1\rangle \otimes |\gamma^1\rangle) \otimes |\alpha_i\rangle \otimes |\beta\rangle. \tag{43}$$

The interaction between the quantum system and the final measurement apparatus and projection onto $|b_j\rangle$ given in (40), leads to the final state of the PPSE:

$$|B; t_b\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle + |c_2^1\rangle) \otimes |\gamma^0\rangle \otimes |\alpha_i\rangle \otimes |\beta_j\rangle. \tag{44}$$

Therefore from (8), $\mathrm{Prob}_f[c^0|A, \{\gamma^0, \gamma^1\}, B] = 1$ in agreement with (A6) of Shimony [14].

Time symmetry is dealt with in Shimony [14] by merely interchanging the initial and final state in (39) and (40) so that the new initial state $|a_i\rangle = (|c_0^0\rangle + |c_2^1\rangle)/\sqrt{2}$ and the new final state $|b_j\rangle = (|c_0^0\rangle + |c_1^1\rangle)/\sqrt{2}$. One then finds

$$|A; t_c\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle \otimes |\gamma^0\rangle + |c_3^1\rangle \otimes |\gamma^1\rangle) \otimes |\beta\rangle \otimes |\alpha_i\rangle \tag{45a}$$

$$|A; t_b\rangle = \frac{1}{\sqrt{2}}(|c_0^0\rangle \otimes |\gamma^0\rangle + |c_1^1\rangle \otimes |\gamma^1\rangle) \otimes |\beta\rangle \otimes |\alpha_i\rangle \tag{45b}$$

$$|B; t_b\rangle = \frac{1}{\sqrt{2}}|b_j\rangle \otimes (|\gamma^0\rangle + |\gamma^1\rangle) \otimes |\alpha_i\rangle \otimes |\beta_j\rangle \tag{45c}$$

so that from (8), $\mathrm{Prob}_f[c^0|A;\{\gamma^0,\gamma^1\},B]=1/2$ in agreement with (A8) of Shimony [14].

As mentioned, the orthodox view is that time symmetry should tested by both the interchanging *and* time-reversing the states. Assuming the Hamiltonian in the example is time symmetric, the time-reversal operator $\hat{\Theta}$ can be found from the condition $\hat{U}\hat{\Theta}=\hat{\Theta}\hat{U}^\dagger$ and in the same $|c^0\rangle$, $|c_i^1\rangle$ basis

$$\hat{\Theta}=\frac{1}{\sqrt{3}}\begin{bmatrix}\sqrt{3} & 0 & 0 & 0\\ 0 & 1 & r & 1\\ 0 & r & 1 & 1\\ 0 & 1 & 1 & r\end{bmatrix}K \tag{46}$$

where $r=-(1+i\sqrt{3})/2$ and K is the operator which takes the complex-conjugate of complex numbers.

Therefore for consideration of the time-reversal of the original PPSE specified by $|a_i\rangle$ and $|b_j\rangle$ given in (39) and (40), the new initial state of the quantum system is

$$|\tilde{a}_i\rangle=\hat{\Theta}|b_j\rangle=\frac{1}{\sqrt{6}}\left[\sqrt{3}|c_0^0\rangle-\frac{1}{2}(1+i\sqrt{3})|c_1^1\rangle+|c_2^1\rangle+|c_3^1\rangle\right] \tag{47}$$

and the new final state of the quantum system is

$$|\tilde{b}_j\rangle=\hat{\Theta}|a_i\rangle=\frac{1}{\sqrt{6}}\left[\sqrt{3}|c_0^0\rangle+|c_1^1\rangle-\frac{1}{2}(1+i\sqrt{3})|c_2^1\rangle+|c_3^1\rangle\right]. \tag{48}$$

One then finds that

$$|A;t_c\rangle=\frac{1}{\sqrt{6}}\left[\sqrt{3}|c_0^0\rangle\otimes|\gamma^0\rangle+\left(|c_1^1\rangle-\frac{1}{2}(1+i\sqrt{3})|c_2^1\rangle+|c_3^1\rangle\right)\otimes|\gamma^1\rangle\right]\otimes|\alpha_i\rangle\otimes|\beta\rangle \tag{49a}$$

$$|A;t_b\rangle=\frac{1}{\sqrt{6}}\left[\sqrt{3}|c_0^0\rangle\otimes|\gamma^0\rangle+\left(|c_1^1\rangle+|c_2^1\rangle-\frac{1}{2}(1+i\sqrt{3})|c_3^1\rangle\right)\otimes|\gamma^1\rangle\right]\otimes|\alpha_i\rangle\otimes|\beta\rangle \tag{49b}$$

$$|B;t_b\rangle=|\tilde{b}_j\rangle\otimes|\gamma^0\rangle\otimes|\alpha_i\rangle\otimes|\beta_j\rangle. \tag{49c}$$

Using (33), $\mathrm{Prob}_r[c^0|A,\{\gamma^0,\gamma^1\},B]=1$ which shows that the PPSE of the counter-example is time-symmetric on the orthodox view of time symmetry. It differs from Shimony [14] because the time reversal of the states was omitted in Shimony [14].

4 Conclusion

The present work is in agreement with one of the main conclusions of Shimony [14, 15] and [16], namely that the initial and final states of the quantum system that is measured are not sufficient to characterise a PPSE because a PPSE is crucially dependent on the nature of the intermediate measuring process(es).

The second conclusion relates to time symmetry and PPSEs. Whether or not time symmetry applies to PPSE's hinges of course on what is meant by the term "time symmetry". Shimony [14] has considered two possible meanings of that term for the particular case of a PPSE. The two meanings seem to be motivated by the claim that was being assessed, but ultimately rejected, in Shimony [14], namely that the initial and final states of the quantum system that is measured are sufficient alone to characterise a PPSE, in which case either meaning would be plausible and physically appealing. The conclusion was that a PPSE fails to satisfy time symmetry on either account.

Once one rejects the idea that the initial and final states of the quantum system are sufficient to characterise a PPSE, there is no reason not to adopt the conventional meaning that "time symmetry" is equivalent to motion-reversal invariance. We have confirmed that PPSE's satisfy time symmetry for the counter-examples of Shimony [14] if one adopts that definition. That conclusion is in agreement with previous conclusions about the time symmetry of PPSE's [4, 5].

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